Lab Manual for B.Sc. Mathematics practical using FOSS (Sci lab/Maxima) for 1st semester.

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Contents

1 Introduction to Scilab

Scilab is a free and open-source software which provides a powerful environment for engineering and scientific computation. It is funded by Scilab Enterprises. It can be used as a supplement to the class room activity to enhance learning as well as solving complex problems with ease.
It can be downloaded from `http://www.scilab.org/download`. Some materials for Scilab can be found on `http://wiki.scilab.org/Tutorialsarchives`

Scilab console

Scilab console is a GUI and hence it can used analogous to a calculator as well as for programming purpose.

ALGEBRA I (Matrices)

2 Introduction to Scilab and commands connected with matrices.

Typical Examples:

1. Type $32 + 5 * 3.19 - 4.651/2.2302 + 3.86^3$ after $-- >$ and press *Enter Key*.
   $-- >$ is the command prompt. Use ^ for raising a number to a given index. Scilab follows the usual rule of precedence for mathematical operations. Appropriate brackets are to be used if necessary. Try out some more expressions to verify the same.

2. Assign values to three variable x, y and z and evaluate the expressions involving the three of them.
   The variable name can be alpha-numeric. The 1^{st} character should be a letter. Scilab is case sensitive. Eg. mat1, matrix123 etc. This comes in handy when certain values have to be repeatedly used.

3. Notice the difference when you enter

$$height = 5.8$$

$$theta = 25.4;$$

4. Identify the difference between $\sin(x)$ and $\text{sind}(x)$.

5. To evaluate $\sin x + \cos 2x - \text{cosec}3x + \sec 5x + 4 \cot 8x$ at $x = 3$ type
   $\sin(x) + \cos(2 * x) - \csc(3 * x) + \sec(5 * x) + 4\text{cotg}(8 * x)$.

6. Built in functions for $tan^{-1}(5x) + cot^{-1}(3x^2)$ are
   atan$(5 * x)$+acotg $(3 * x\,\hat{}\,2)$.

7. $\tan(\frac{4\pi}{3}) + \tan(\frac{5\pi}{6})$ can be obtained by the commands
   $\tan(4 * \%pi/3) + \tan(5 * \%pi/6)$.

   Make use of the help browser for the description and syntax of the two functions. Availability of other built-in functions can be searched on the *Help Browser*.

Exercises

1. If $a = 15$ and $b = 225$ find
   (i) $a+b$ (ii) ab (iii) $a-b$ (iv) a/b (v) a^b
   (vi) b^a (vii) $(a^b)^a$ (viii) a^{a^2} (ix) $(a+b)^{a+b}$ (x) $(ab)^{ab}$.

2. If $a = 25$ and $b = 10$ find
   (i) $\sqrt{a}$ (ii) $\sqrt{ab}$ (iii) $\sqrt{a+b}$ (iv) $\sqrt{(a^2+b^2+2)}$ (v) $\sqrt{a+b+3ab}$
   (vi) $\sqrt[3]{a+b}$ (vii) $a^{(3/2)}$ (viii) $a^{5/2}+b^{2/5}$ (ix) $a^{a^{1/4}}$ (x) $a^{3/5}+a^{5/3}$.

3. Evaluate (i) $log_{10}a$ (ii) $log_e a$ (iii) e^a (iv) e^{a^2} (v) e^{e^a}
   (vi) $\sin a$ (vii) $\sin^{-1} a$ (viii) $(\sin^{-1} a)^2$ (ix) $\sin^2 a+\cos^2 a$ (x) $\csc^2 a+\sin^2(1/a)$. Take $a = 53$.

4. If $a = 43$ and $b = 25$ determine the following.
   (i) $\sin(a+b)$ (ii) $\sin a\cos b+\cos a\sin b$ (iii) $\sin(a-b)$
   (iv) $\sin a\cos b-\cos a\sin b$ (v) $\tan(a+b)$ (vi) $\tan^{-1}(ab)$
   (vii) $\sin(2a)$ (viii) $\cos 3a+\sin 2a$ (ix) $\cos 5a+\sin 2b-\csc^2(5a+2b)$ (x) $\csc^2 a^2$.

5. Verify

   (i) $\sin(a+b) = \sin a\cos b+\cos a\sin b$

   (iii) $\tan(a+b) = \dfrac{\tan a+\tan b}{1-\tan a\tan b}$

   (v) $\sin^2 a+\cos^2 a = 1$

   (vi) $\cos 2a = 2\cos^2 a-1 = 1-2\sin^2 a$

   (vii) $\sin^2 a+\cos^2 b \neq 1$

   (ix) $cos(\pi/2)+i\sin(\pi/2) = e^{i\pi/2}$

   Take $a = 10$ and $b = 20$.

6. Find the values in degree,
   (ii) $\sin(\pi/2)$ (ii) $\sin(3\pi/2)$ (iii) $\csc(2/\pi)$ (iv) $\cot\frac{55\pi^2}{66}$
   (v) $64\tan^2\frac{3\pi^2}{5}+38\csc^2\frac{3\pi^2}{5}$ (vi) $\csc^3\frac{437\pi^3}{567}$ (vii) $\sec(\frac{\pi^2}{2}+\frac{34\pi^5}{7})$
   (viii) $\cot^3(\pi/2)+\sec^3(3\pi/2)$ (ix) $2\sin^2\pi+\cos^2\frac{7\pi}{2}$ (x) $\cos\frac{3\pi}{5}+\sec\frac{3\pi}{5}$.

7. Create a polynomial $x^4-3x^3+1 = 0$ find real roots and coefficients of the polynomial.

3 Computations with matrices.

Scilab being a matrix environment, treats all entries as a matrix. A matrix can be entered by typing [1 2; 3 1] at the command prompt. Note the role of ; inside and outside the square brackets. In Scilab they are many built in functions to create different matrices that is listed below
eye(n) - identity matrix of order n
zeros(n) - zero matrix of order n
ones(n) - unit matrix
diag() - diagonal matrix
rand()- random matrix.
Few more library or built in functions
inv(A) - for inverse of matrix A
det(A)- for determinant of A
size(A)- order of A
rank(A)- rank of A
spec(A)- Eigenvalue(spectrum) of A
min(A)-minimum value(elements) in the matrix
max(A)- maximum value in the matrix A
prod(A)- for product of each elements.
sum(A)- for sum of all the elements in the matrix A.
length(A)-number of elements of A.
One can add,subtract,multiply and divide two matrices(follow rules of matrices).
For example suppose $A = \begin{bmatrix} 1 & 2 & 1 \\ 4 & -1 & 1 \\ 1 & 6 & 9 \end{bmatrix}$ and $B = \begin{bmatrix} -8 & 2 & 1 \\ 4 & -1 & 3 \\ 4 & 7 & 9 \end{bmatrix}$.
$A + B$ gives addition two matrices. Similarly for subtraction,multiplication and division $A - B, AB, A/B$ commands used.
$2A$ produces scalar multiplication.
$A.B$ for elementary multiplication.
AA produces product of A with A that is A^2.
$A(2, 2)$ refers for element from second row second column(as like matrix representation).
$A(4)$ refers for 4th element(counted from row-wise)
$a = A(1 : 2, ;)$ creates submatrix 1st row to 2nd row includes all columns.

Exercises

1. Create matrices $A = \begin{bmatrix} 2 & 3 & 1 \\ 4 & -1 & 3 \\ 1 & 6 & 8 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 3 & -2 \\ 8 & 2 & 1 \\ 1 & -1 & 6 \end{bmatrix}$.

2. Create a matrix P of order 3×4 by taking random elements and then find transpose of P.

3. Create matrices $p = \begin{bmatrix} 1 & 5 & 6 \\ 7 & 8 & 10 \\ 0 & 1 & 5 \end{bmatrix}$ and $q = \begin{bmatrix} 7 & 8 & 10 \\ 15 & 1 & 2 \\ 3 & 7 & 5 \end{bmatrix}$. Compute
   (i) $p + q$ (ii) pq (iii) $p - q$ (iv) p^{-1} (v) p^2
   (vi) $3p + 4q$ (vii) $5p^2 + 7q^2$ (viii) $p \cdot q$(elementwise multiplication)
   (ix) $p' + q'$ (x) $3pq' + 5qp'$.

4. In Scilab construct $A = \begin{bmatrix} 5 & -3 & 2 \\ -6 & 10 & 7 \\ 2 & 4 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 4 * 2 & 2/3 \\ 1/205 & 5 - 7.8 \\ 0.0001 & (9+8)/50 \end{bmatrix}$.
   Using Scilab commands display the following
   (i) a_{23} (ii) b_{32} (iii) $row_1(A)$ (iv) $col_1(A)$ (v) $row_2(B)$.

5. Create matrices $A = \begin{bmatrix} 1 & 1/2 \\ 1/3 & 1/4 \\ 1/5 & 1/6 \end{bmatrix}$, $B = \begin{bmatrix} 5 & -2 \end{bmatrix}$ and $C = \begin{bmatrix} 4 & 4/5 & 10/7 \\ 1 & 5 & 6 \end{bmatrix}$. Using scilab command, compute the following if possible, otherwise give reason why its not possible
   (i) $A * C$ (ii) $A \cdot * C$ (iii) $A + C'$ (iv) $B * A - C' * A$
   (v) $(2 + C - 6 * A') * B'$ (vi) $A * C - C * A$ (vii) $A * A' + C' * C$.

6. Create
   (i) 5×4 unit matrix.
   (ii) 10×11 zero matrix.
   (iii) 14×14 diagonal matrix.
   (iv) 15×15 random matrix.
   (v) 16×16 scalar matrix.
   (vi) 6×6 identity matrix.
   (vii) 6×6 tridiagonal matrix.
   (viii) 4×4 diagonal matrix with diagonal elements equal to five.

7. Create a 4×4 square matrix with random elements from 0 to 36 then find

   (i) the square of the matrix.

   (ii) element by element square.

   (iii) trace of the matrix.

   (iv) eigenvalue of the matrix.

   (v) maximum value.

   (vi) minimum value.

   (vii) sum of each column.

   (viii) sum of each row.

   (ix) product of all the elements.

   (x) display 3rd row.

4 Row reduced echelon form and normal form.

Scilab commands:

Example: Create $A = \begin{bmatrix} 1 & 2 & 3 \\ 5 & 7 & 8 \\ 2 & 1 & 1 \end{bmatrix}$.

Find the row reduced echelon form and normal form of A.

$A = [1\ 2\ 3; 5\ 7\ 8; 2\ 1\ 1]$
$[m, n]$=size(A);
a=rref(A);
mprintf("row reduced echelon form of A is $\backslash n$");
disp(a);
for $i = 1 : m$
  for $j = i + 1 : n$
    $a(i, j) = a(i, j) - a(i, j) * a(i, i)$;
    end
  end
mprintf("normal form of A is $\backslash n$");
disp(a);

Exercises

1. Create a matrix $A = \begin{bmatrix} 5 & 6 & 7 \\ 1 & 2 & 3 \\ 7 & 3 & 5 \end{bmatrix}$.

   (i) Write a command to find row reduced echelon form of A.

   (ii) Find normal form of A, adjoint of A, inverse of A and eigenvalues of A.

2. Create a matrix, $A = \begin{bmatrix} 5 & 2 & 1 \\ 6 & 7 & 8 \\ 5 & 2 & 1 \end{bmatrix}$

   (i) Find row reduced echelon form of A manually compare it with Scilab command rref(A).

   (ii) Find normal form of A.

   (iii) Construct lower triangular matrix from A.

   (iv) Construct upper triangular matrix from A.

   (v) Construct submatrix $\begin{bmatrix} 6 & 8 \\ 5 & 1 \end{bmatrix}$ from A.

   (vi) Construct diagonal matrix having principal diagonal elements 5,7 and 1.

(vii) Construct submatrix $\begin{bmatrix} 5 & 1 \\ 5 & 1 \end{bmatrix}$ from A.

(viii) Construct matrix $\begin{bmatrix} 0 & 2 & 0 \\ 1 & 0 & 3 \\ 0 & 3 & 0 \end{bmatrix}$

from A using any operation (row or column).

3. Find the rank of the matrix $A = \begin{bmatrix} 6 & 1 & 3 & 8 \\ 4 & 2 & 6 & -1 \\ 10 & 3 & 9 & 7 \\ 16 & 4 & 12 & 15 \end{bmatrix}$.

4. Reduce to normal form and hence find its rank $A = \begin{bmatrix} 1 & 1 & -1 \\ 2 & -3 & 4 \\ 3 & -2 & 3 \end{bmatrix}$.

5. Create a matrix, $M = \begin{bmatrix} -7 & 0 & 0 & 0 \\ 0 & -7 & 0 & 0 \\ 0 & 0 & 0 & -7 \end{bmatrix}$ using Scilab command $diag()$.

6. Using command $diag()$ construct matrix $\begin{bmatrix} 0 & 0 & -2 & 0 & 0 \\ 8 & 0 & 0 & -2 & 0 \\ 0 & 8 & 0 & 0 & -2 \\ 9 & 0 & 8 & 0 & 0 \end{bmatrix}$.

   Note: Use only $diag()$ commands in single line construct the above matrix.

5 Establishing consistency or otherwise and solving system of linear equations.

Scilab commands:

Example: Create $A = \begin{bmatrix} 1 & 2 & 3 \\ 5 & 7 & 8 \\ 2 & 1 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 \\ 5 \\ 6 \end{bmatrix}$.
Find inverse of A and solve $AX = B$.
For finding inverse of A use the command $\backslash inv(A)$.
For finding X use the command $A\backslash B$ or use built in function inv().
i.e., $X = \text{inv}(A) * B$.

Example 2: Solve $1x + 5y + z = 10$;$5x + y + 3z = 5$;$x + 7y + 4z = 15$.
Create a matrix A whose elements are coefficients of the above equations. That is $A = [1\ 5\ 1; 5\ 1\ 3; 1\ 7\ 4]$. Then create a column vector B whose elements are right hand side of the equations. That is $B = [10; 5; 15]$. Now to find X use the command $A\backslash B$.

Exercises

1. Create $A = \begin{bmatrix} 11 & 12 & 23 \\ 5 & 7 & 8 \\ 12 & 1 & 13 \end{bmatrix}$ and $B = \begin{bmatrix} 101 \\ 5 \\ 60 \end{bmatrix}$. Type the Scilab command,
   (i) A/B (ii) $inv(A) * B$ and (iii) $A\backslash B$.
   Note down the outputs and explain the difference of it.

2. Solve $AX = B$ where $A = \begin{bmatrix} 52 & 71 & 80 \\ 95 & 24 & 63 \\ 15 & 96 & 21 \end{bmatrix}$ and $B = \begin{bmatrix} 105 \\ 71 \\ 64 \end{bmatrix}$.

3. Solve

   (i) $3x + 5y + 10z = 5$
   $2x + 4y + 5z = 6$.

   (ii) $5x + 2y = 10$
   $10x + 4y = 0$.

   (iii) $x + 4y + 5z = 3$
   $2x + 6y + 8z = 5$
   $3x + 7y + 22z = 7$.

   (iv) $3x_1 + x_2 + 5x_3 = 3$
   $2x_2 + x_3 = 7$
   $x_1 + x_3 = 9$.

   are these consistence? Explain.

4. Solve

(i) $10x + 5y + 2z = 10$; $5x + 2y + 3z = 5$; $6x + 7y + 4z = 5$.

(ii) $5x_1 + 3x_2 + 4x_3 = 2$; $3x_1 + 2x_2 + 5x_3 = 8$; $4x_1 + 3x_2 + x_3 = 9$.

(iii) $2a - b = 10$; $3a + 2c = 5$; $4b + 5c = 15$.

(iv) $15a_1 + 23a_2 + 51a + 3 = 27$; $3a_1 + 25a_3 = 38$; $5a_2 + 23a_3 = 49$.

(v) $2p + 4q + 5 = 0$; $7p + 5q - 8 = 0$.

6 Introduction to Maxima

Maxima is an open-source computer algebra system(CAS) for solving typical Calculus problems. The latest version of this document can be found at http://maxim.sourceforge.net/documentation.html. Leon Q. Brin is a professor at Southern CT State University, New Haven, CT U.S.A. Feedback is welcome at BrinL1@southernct.edu.

7 Introduction to Maxima and commands for derivatives and n^{th} derivatives.

Maxima commands:

Example: If $y(x) = \sin^3 x$, find u_x, u_{xx} and u_{xxx}.
$y : \sin\hat{}\, 3x;$
$y1 : diff(u, x);$
$y2 : diff(y1, x);$ or $y2 : diff(y, x, 2);$

Exercises

1. Find differentiation of $2x^2 + x^3 + \sin x$.

2. Differentiate (i) $\sin x \cdot \cos 2x$ (ii) $\sin^2 x \cdot \cos 2x$
   (iii) $x^3 + 2 \sin x \cdot \sec x$ (iv) $e^{2x} + x^2 + 1$ (v) $logx + e^{5x} + 1$.

3. Differentiate twice with respect to x
   (i) $\sin x \cdot \cos 3x$ (ii) $(x^2 + 1)^5$ (iii) $(1 - x)^4$ (iv) $logx$ (v) e^{2x^2}.

4. Apply 4th order differentiation w.r.t. x to the following
   (i) e^{2x} (ii) $4e^{3x^2}$ (iii) $\sin(3x) + 7\cos(5x)$ (iv) $\dfrac{1}{(1-x)^5}$ (v) $1 + x^3 + x^5$ (vi) $log(x)$ (vii) $log4x + 3\cos 8x$ (viii) $(x^2 + 1)^10$ (ix) $\cos^4(7x)$ (x) $\tan^{-1} x$.

5. Solve $\dfrac{d^3}{dt^3}(x^5 + 3x^4 + 3x + 7)$.
   (i) Find the value of derivative at $x = 9.7$.
   (ii) Find the highest degree of derivative.
   (iii) Find the root of equation(derivative).

(iv) Find eigenvalue of equation(derivative).

(v) Find the coefficients of derivative.

6. Find u_x, u_{xx}, u_{xxx} and u_{xxxx} for the following

(i) $u = \sin(2x) + \cos(3x)$

(ii) $u = e^{3x}$

(iii) $u = log(x)$

(iv) $u = \sin x \cdot \cos 3x$

(v) $u = \tan x$

7. Find $\dfrac{d^n}{dx^n}(\sin 2x)$ by taking different values of n.

8 Scilab and Maxima commands for plotting functions.

Maxima commands:

Example 1: Plot a graph for $y(x) = \sin x, x$ ranges from 0 to 2π.
$wxplot2d(\sin(t), [t, 0, 2 * \%pi])$.

Example 2: Plot a graph for $y(x) = \sin x, x$ ranges from 0 to 2π.
$wxplot2d([\sin(t), \cos(t)], [t, 0, 2 * \%pi])$.

Example 3: Plot graph for discrete points. Label x and y axis. Also take point type as asterisk and use legend,color.
$xy : [[10, .6], [20, .9], [30, 1.1], [40, 1.3], [50, 1.4]]$
$plot2d\,([[discrete, xy], 2 * \%pi * sqrt(l/980)], [l, 0, 50]\,,$
$[style, points, lines],$
$[color, red, blue],$
$[point\_type, asterisk],$
$[legend, "experiment", "theory"],$
$[xlabel, "pendulum's length(cm)"],$
$[ylabel, "period(s)"])$

Example 4: Plotting three dimensional graphs.
$plot3d(2\hat{}(-u\hat{}2 + v\hat{}2), [u, -3, 3], [v, -2, 2])$

Example 5: Plotting 3D graphs by mentioning grid points(mesh points).
$plot3d\,(log(x\hat{}2 * y\hat{}2), [x, -2, 2], [y, -2, 2]$
$[grid, 29, 29],$
$[palette, get\_plot\_option(palette, 5)])$.

Exercise

1. Plot a graph for $y(x) = x^2 + 1$, x ranges from 0 to 5.

2. If $y(x) = x^2 + 2x + 1$, plot a graph x vs y $\forall x \in [-2, 2]$. Point out the root by using the figure.

3. If $y(x) = \sin x$. Plot a graph by taking interval $[0, 2\pi]$.
   (i) Take 50 points between 0 and 2π.
   (ii) Label x- axis and y- axis.
   (iii) Name(title) figure as sine curve.
   (iv) Plot the curve in dotted line.
4. Plot curves $y_1 = \sin x$, $y_2 = \sin 2x$,$y_2 = \sin 3x$, $x \in [0, 2\pi]$.
5. Apply dotted line for $\sin x$, dashed line for $\sin 2x$, thick line for $\sin 2x$.
6. Plot $y_1 = \sin x, y_2 = \cos x$ from the interval $[0, 2\pi]$.
   (i) Label x and y axis.
   (ii) Give title to figure.
   (iii) Use red color for the curve $y_1 = \sin x$ and green color for $y_2 = \cos x$.
7. Plot 3D graph for
   (i) equation of line with the interval 1 to 5 and range 0.05.
   (ii) plane represented by the equation $ax + by + cz + d = 0$ with the interval 0 to 5 and range 0.5
   (iii) plane parallel to y-axis with interval 0 to 5 and range 0.5
   (iv) plane parallel to x-axis with interval 0 to 5 and range 0.5
   (v) plane parallel to xy-axis with interval 0 to 5 and range 0.5
   (vi) sphere using param 3D
   (vii) circle with the interval 0 to 2π and range $\pi/16$.
   (viii) cylinder with the interval 0 to 2π and range $\pi/16$.
8. Plot a discrete set of points defining x and y coordinates as $[10, 20, 30, 40, 50]$ and $[0.6, 0.9, 1.1, 1.3, 1.4]$.
9. Plot the following

(i) $2^{(-u^2+v^2)}$, $u \in [-3,3]$, $v \in [-2,2]$.

(ii) $log(x^2y^2)$, $x \in [-2,2], y \in [-2,2], z \in [-8,4]$ color it as magenta and blue.

(iii) $2^{(-x^2+y^2)}, \dfrac{4\sin(3(x^2+y^2))}{x^2+y^2}$, $x \in [-3,3], y \in [-2,2]$.

(iv) $x^2+y^2, x \in [-4,4], y \in [-4,4]$.

(v) x^2-1, $x \in [-3,3], y \in [-2,10]$.

(vi) e^{2s}, $s \in [-2,2]$.

(vii) $r\sin t, r\cos t$ where $r = e^{\cos t} - 2\cos 4t - \sin(t/12)^5$, $t \in [-8\pi, 8\pi]$.

10. Plot Klein bottle,
$r_1 = 5\cos x(\cos(x/2)\cos y + \sin(x/2)\sin 2y + 3) - 10$
$r_2 = -5\sin x(\cos(x/2)\cos y + \sin(x/2)\sin 2y) + 3$
$r_3 = 5(-\sin(x/2)\cos y + \cos(x/2)\sin 2y)$
Take $x \in [-\pi,\pi], y \in [-\pi,\pi]$ and grid points $[40,40]$.

9 n^{th} derivative without Leibnitz rule.

Maxima commands:

Example 1: Find n^{th} derivative of $F = \cos x\ \cos 2x\ \cos 3x$ by taking appropriate values of n.

$F : cos(x) * cos(2 * x) * cos(3 * x);$

$yn : diff(F, x, 3);$

Note: In this case $n = 3$.

Exercises

1. Find 6^{th} derivative of the following

   (i) $\sin(x)$ (ii)e^{ax} (iii)$xlog(x)$

   (iv) $alog(\dfrac{1}{1+x^2})$ (v)$\tan x \cos x$ (vi)5^{3x}

   (vii)e^{-4x} (viii)$\sin(2x-1)$ (ix)$\cos(1-3x)$

   (x)$\dfrac{1}{4x-1}$ (xi)$\dfrac{1}{(2x-3)^6}$ (xii)$(6x+5)^7$

   (xiii)$\tan^{-1}\left(\dfrac{x}{a}\right)$.

2. Find n^{th} derivative of

   (i) $\tan^{-1}\left(\dfrac{1+x}{1-x}\right)$ (ii)$log(x^2-4)$

   (iii)$\cos^2(4x)$ (iv)$\dfrac{x+1}{x^2-4}$

   (v)$\sin 3x \cos x$ (vi)$cos^3 2x$

   (vii)$\dfrac{3x}{(2x+1)(x-1)}$ (viii)$e^{-2x}\cos 3x$

   (ix)$\cos x \cos 2x \cos 3x$ (x)$\dfrac{x}{(x-a)(x-b)(x-c)}$

   Take n as 2,5,6.

10 n^{th} derivative with Leibnitz rule.

Leibnitz rule:

$$(uv)_n = nC_0u_nv + nC_1u_{n-1}v_1 + nC_2u_{n-2}v_2 + \cdots + nC_ru_{n-r}v_r + \cdots + nC_nuv_n.$$

Maxima commands

$u : x^2;$
$v : sin(x);$
$diff(u, x)$
$diff(u, x, 2);$
$diff(v, x, 3) * u + 3 * diff(v, x, 2) * diff(u, x) + 3 * (3 - 1) * diff(v, x, 1) * diff(u, x, 2);$

Note: Differentiate u until to get constant term. Then take n as the order of differentiation. In above example order to 2.

Exercises

1. Find the nth derivatives of the following

   (i) $x^2 \cos x$ (ii)$e^{2x} \sin 3x$ (iii) $\sin 4x \cos 2x$

   (iv) $log(5x - 1)$ (v) $e^{2x} \sin h3x$ (vi) $\cos 2x \cos 3x$

   (vii)$\cos 2x \sin 3x$ (viii) x^2e^x (ix) x^7e^{3x} (x) $x^4 \sin 2x.$

2. Find the n th derivative of

(i) $x^2e^x \cos x$

(ii) $x^3 \cos x$

(iii) x^4logx

(iv) e^xlogx

(v) $x^4 \tan x.$

11 Obtaining partial derivative of some standard functions.

Maxima commands:

Example: If $u(x,y) = e^{\frac{x}{y}}$, find $u_x, u_{xx}, u_y, u_{yy}, u_{xy}$ and u_{yx}.

$u : e\hat{}(x/y);$

$ux : diff(u,x);$

$uy : diff(u,y);$

$uxx : diff(ux,x);$ or $uxx : diff(u,x,2);$

$uyy : diff(uy,y);$

$uxy : diff(ux,y);$

$uyx : diff(uy,x);$

Exercises:

1. If $f(x,y) = tan^{-1}\left(\frac{x}{y}\right)$, find (i) $\frac{\partial f}{\partial x}$ (ii) $\frac{\partial f}{\partial y}$ (iii) $\frac{\partial^2 f}{\partial x \partial y}$ (iv) $\frac{\partial^2 f}{\partial y \partial x}$.

2. If $f(x,y) = x^y + y^x$, find (i) f_x (ii) f_{xx} (iii) f_y (iv) f_{yy} (v) f_{xxx} (vi) f_{xxy} (vii) f_{xyy} (viii) f_{yyy} (ix) f_{yxy} (x) f_{yyx}.

3. If $u = x^2 \tan^{-1}\left(\frac{y}{x}\right)$, show that (i) $u_{xy} = u_{yx}$ (ii) $u_{yx} - u_{xy} = 0$.

4. If $z = (1 - 2xy + y^2)^{-1/2}$, show that $x\frac{\partial z}{\partial x} - \frac{\partial z}{\partial y} = y^2 z^3$.

5. If $x = r\cos\theta$ and $y = r\sin\theta$, show that

   (i) $\frac{\partial r}{\partial x} = \frac{x}{\sqrt{x^2 + y^2}}; \frac{\partial r}{\partial y} = \frac{y}{\sqrt{x^2 + y^2}}$.

   (ii) $\frac{\partial \theta}{\partial x} = -\frac{y}{x^2 + y^2}; \frac{\partial \theta}{\partial y} = \frac{x}{x^2 + y^2}$.

   (iii) $\left(\frac{\partial r}{\partial x}\right)^2 + \left(\frac{\partial r}{\partial y}\right)^2 = 1$.

   (iv) $\frac{\partial^2 \theta}{\partial x^2} + \frac{\partial^2 \theta}{\partial y^2} = 0$.

(v) $\dfrac{\partial^2 r}{\partial x^2} \cdot \dfrac{\partial^2 r}{\partial y^2} = \left(\dfrac{\partial^2 r}{\partial x \partial y}\right)^2.$

6. Express $\dfrac{d^3y}{dx^3} + \dfrac{d^2y}{dx^2} + x^2\dfrac{dy}{dx}$. Find out the highest degree of the derivative.

12 Verification of Euler's theorem, its extension and Jacobian.

Homogeneous function: $u(x,y) = x^n \phi\left(\frac{y}{x}\right)$ or $u(x,y) = y^n \phi\left(\frac{x}{y}\right)$.

Euler theorem: $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = nu$.

Extension of Euler theorem:

$$x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial u}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial y^2} = n(n-1)u.$$

Maxima commands:
Example 1 : If $u = ax^2 + 2hxy + by^2$; verify Euler's theorem.

```
u : a*x^2 + 2*h*x*y + b*y^2;
ux : diff(u, x);
uy : diff(u, y);
euler : x*ux + y*uy;
eulersimply : ratsimp(euler);
fact : factor(eulersimply);
is(fact = 2*u);
```

Note: Run each line by using keys shift+enter

Example 3: If $u = log\left(\frac{x^3 + x^2y - y^2x + 2y^3}{x+y}\right)$ prove that
$x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial u}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial y^2} = -2.$

```
u : log((x^3 + x^2*y - y^2*x + 2*y^3)/(x + y));
ux : diff(u, x);
uxx : diff(u, x, 2);
uyy : diff(u, y, 2);
uxy : diff(ux, y);
eulerext : x^2*uxx + y^2*uyy + 2*x*y*uxy;
ratsimp(eulerext);
```

Example 4 : If $u = x^2 - 2y$ and $v = x + y$ find $J = \dfrac{\partial(u,v)}{\partial(x,y)}$.

J : jacobian($[x\hat{\ }2 - 2 * y, x + y], [x, y]$);
D : determinant(J);

Exercises:

1. If $u = \dfrac{x}{x-y}$ verify Euler's theorem.

2. If $u = xy\sin\left(\dfrac{x}{y}\right)$ verify Euler's theorem.

   i.e., prove $x\dfrac{\partial u}{\partial x} + y\dfrac{\partial u}{\partial y} = nu$. Here $n = 2$.

3. If $u = \dfrac{x}{y}\cos(xy)$ prove that $x\dfrac{\partial u}{\partial x} - y\dfrac{\partial u}{\partial x} = 2u$.

4. If $ax^2 + 2hxy + by^2$ verify Euler's theorem.

5. If $u = \sin^{-1}\left(\dfrac{x^2+y^2}{x+y}\right)$ show that $x\dfrac{\partial u}{\partial x} + y\dfrac{\partial u}{\partial y} = tanu$.

6. If $u = log\left(\dfrac{x^3 + x^2y - y^2x + 2y^3}{x+y}\right)$ prove that

   (i) $x\dfrac{\partial u}{\partial x} + y\dfrac{\partial u}{\partial y} = 2$.

   (ii) $x^2\dfrac{\partial^2 u}{\partial x^2} + 2xy\dfrac{\partial u}{\partial x\partial y} + y^2\dfrac{\partial^2 u}{\partial y^2} = -2$.

7. If $u = tan^{-1}\dfrac{x^2+y^2}{x-y}(x \neq y)$, show that

   (i) $x\dfrac{\partial u}{\partial x} + y\dfrac{\partial u}{\partial y} = \sin 2u$.

   (ii) $x^2\dfrac{\partial^2 u}{\partial x^2} + 2xy\dfrac{\partial u}{\partial x\partial y} + y^2\dfrac{\partial^2 u}{\partial y^2} = (1 - 4\sin^2 u)\sin 2u$.

8. If $u = 2xy$ and $v = x^2 - y^2$ find $J = \dfrac{\partial(u, v)}{\partial(x, y)}$.

9. If $x = r\cos\theta, y = r\sin\theta$ find $J = \dfrac{\partial(x, y)}{\partial r, \theta}$ and $J' = \dfrac{\partial(r, \theta)}{\partial(x, y)}$. Verify $JJ' = 1$.

INTEGRAL CALCULUS

13 Maxima commands for reduction formula with or without limits.

Scilab code:

Reduction formula for integral 0 to $pi/2$ of $(\sin(x))^n$.

function[p]=$f(n)$
$p = 1;$
$k = \%pi/2;$
for $i = 1 : 2 : n - 1$
if (modulo$(n, 2)$ == 0) then
$p = ((n - i)/(n - (i - 1))) * k;$
$k = p;$
else
$p = ((n - i)/(n - (i - 1))) * p;$
end
end
mprintf("$I(\%d) = \%f$", n, p);
endfunction

1. Using reduction formula find,
   (i) $\int \sin^7 x dx$ (ii) $\int \cos^7 x dx$ (iii) $\int \sec^5 x dx$
   (iv) $\int \sin^5 x dx$ (v) $\int \cos^4 x dx$.

2. Evaluate
   (i) $\int_0^{\pi/2} \sin^6 x dx$ (ii) $\int_0^{\pi/2} \sin^9 x dx$
   (iii) $\int_0^{\pi} \cos^5 x dx$ (iv) $\int_0^{3\pi} \sin^6(x/6) dx$.

3. Evaluate
   (i) $\int_0^{\pi/4} \sec^3 x dx$ (ii) $\int_0^{\pi/4} \sec^6 x dx$
   (iii) $\int_{\pi/4}^{-\pi/4} \sec^5 x dx$ (iv) $\int_{\pi/2}^{\pi/4} \csc^7 x dx$.

4. Evaluate
   (i) $\int \cos^3 x dx$ (ii) $\int \sin^6 x dx$ (iii) $\int \csc^5 x dx$
   (iv) $\int \sec^7 \theta d\theta$ (v) $\int \tan^4 x dx$.

GEOMETRY

14 Implementing vector form of line.

Vector equation of a straight line passing through a given point and parallel to a given vector.
$\overrightarrow{r} = \overrightarrow{a} + t\overrightarrow{v}$
where $\overrightarrow{a}$ is position vector and $\overrightarrow{v}$ is given vector.
Cartesian form:
$\dfrac{x - x_1}{x_2 - x_1} = \dfrac{y - y_1}{y_2 - y_1} = \dfrac{z - z_1}{z_2 - z_1}$

Maxima commands
Vector form:
$a : 1 * i + 2 * j - 4 * k;$
$v : 2 * i + 3 * j + 2 * k;$
$r : a + t * v;$

Cartesian form:
$x1 : 5; y1 : 2; z1 : 7;$
$x2 : 3; y2 : 5; z2 : 2;$
$(x - x1)/(x2 - x1) = (y - y1)/(y2 - y1);$
$(x - x1)/(x2 - x1) = (z - z1)/(z2 - z1);$

1. Find the equation of the line in vector form of the following and plot graph for each.
   (i) $\overrightarrow{a} = (2, 1, -3)$ $\overrightarrow{v} = \hat{i} - 2\hat{j} + 3\hat{k}$.
   (ii) $\overrightarrow{a} = (1, -1, 1)$ $\overrightarrow{v} = \hat{i} - \hat{j} + 3\hat{k}$.
   (iii) $\overrightarrow{a} = (0, 2, -3)$ $\overrightarrow{v} = 2\hat{i} - \hat{j} + \hat{k}$.
   (iii) $\overrightarrow{a} = (0, 2, -3)$ $\overrightarrow{v} = 2\hat{i} - \hat{j} + \hat{k}$.
   (iv) $\overrightarrow{a} = (0, 2, -1)$ $\overrightarrow{v} = \hat{i} - 2\hat{j} + \hat{k}$.
   (v) $\overrightarrow{a} = (2, 3, -5)$ $\overrightarrow{v} = 2\hat{i} - 3\hat{j} + 4\hat{k}$.
   (vi) $\overrightarrow{a} = (1, 2, -3)$ $\overrightarrow{v} = \hat{i} - \hat{j} + \hat{k}$.
   (vii) $\overrightarrow{a} = (3, -1, 9)$ $\overrightarrow{v} = 3\hat{i} - 7\hat{j} + \hat{k}$.
   (viii) $\overrightarrow{a} = (1, 0, 0)$ $\overrightarrow{v} = \hat{i} - 2\hat{j} + \hat{k}$.

(ix) $\overrightarrow{a} = (1, 0, -1)$ $\overrightarrow{v} = 3\hat{\imath} - 4\hat{\jmath}$.

(x) $\overrightarrow{a} = (1, 2, -3)$ $\overrightarrow{v} = 2\hat{\imath} - 3\hat{\jmath} + 8\hat{k}$.

[Plot graph of all simultaneously and distinguish them by using different color].

2. Find the Cartesian equations of the line joining the points in the following cases.

   (i) $A(1, -2, 1)$ and $B(1, 3, -2)$.

   (ii) $A(2, -1, 4)$ and $B(1, 1, -2)$.

   (iii) $A(1, -8, 1)$ and $B(1, 2, -3)$.

   (iv) $A(0, -1, 5)$ and $B(2, 1, -2)$.

   (v) $A(1, -4, 1)$ and $B(9, -1, 2)$.

15 Implementing vector form of plane.

vector form

$\hat{n} = \frac{\overrightarrow{n}}{|\overrightarrow{n}|}$.

Maxima commands:

Example 1: Find the vector equation of the plane which is at a distances of 4 units from the origin and which is normal to the vector $\left(\hat{i} + 2\hat{j} + 5\hat{k}\right)$.

$n : i + 2 * j + 5 * k;$
$p : 4;$
$n1 : sqrt(1 + 4 + 25);$
$n2 : n/n1;$
$eqn : r.n2 = 4;$

1. Find the vector equation of the plane which is at a distances of 7 units from the origin and which is normal to the vector $\left(\hat{i} + 2\hat{j} - 2\hat{k}\right)$.
2. Find the vector equation of the plane which is at a distances of 3 units from the origin and which is normal to the vector $\left(\hat{i} + 12\hat{j} + 2\hat{k}\right)$.
3. Find the equation of the plane in vector form and plot a graph for each.
   (i) Distance = 2 units from the origin and normal to the vector $(2\hat{i} + 5\hat{j} + \hat{k})$.
   (ii) Distance = 5 units from the origin and normal to the vector $(\hat{i} + \hat{j} + \hat{k})$.
   (iii) Distance = 12 units from the origin and normal to the vector $(3\hat{i} + 7\hat{j} + 2\hat{k})$.
   (iv) Distance = 23 units from the origin and normal to the vector $(2\hat{i} + 5\hat{j} + 4\hat{k})$.
   (v) Distance = 9 units from the origin and normal to the vector $(2\hat{i} + 7\hat{j} + 6\hat{k})$.
   (vi) Distance = 7 units from the origin and normal to the vector $(\hat{i} + 15\hat{j} + 9\hat{k})$.

(vii) Distance $= 5$ units from the origin and normal to the vector $(6\hat{i} + 5\hat{j} + 3\hat{k})$.

(viii) Distance $= 4$ units from the origin and normal to the vector $(5\hat{i} + 5\hat{j} + 5\hat{k})$.

(ix) Distance $= 26$ units from the origin and normal to the vector $(2\hat{i} + 5\hat{j} + 7\hat{k})$.

(x) Distance $= 100$ units from the origin and normal to the vector $(2\hat{i} + 9\hat{j} + \hat{k})$.

4. The vector equation of a plane is $\overrightarrow{r} \cdot (2\hat{i} - \hat{j} + 2\hat{k}) = 9$. Reduce it to the normal form and hence find the length of the perpendicular from the origin to the plane.

16 Glossary terms

GUI - Graphical user interface

Scilab - Science laboratory

CAS - Computer algebra system